MR1306537 (95j:42031) 42C20 35J35 65N30

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★**Wavelet interpolation and approximate solutions of elliptic partial differential equations. (English summary)**

Noncompact Lie groups and some of their applications (San Antonio, TX, 1993), 349–366, *NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci.*, 429, *Kluwer Acad. Publ.*, *Dordrecht*, 1994.

A second-order interpolation theorem for finite energy functions is derived as a locally finite series of Daubechies wavelets. Samples of a sufficiently smooth function are used as coefficients of the fine scale wavelet expansion. The corresponding wavelet interpolation functions converge in the first-order Sobolev norm to the original function. In this context, the main result, while valid on a bounded open set $\Omega \subset \mathbf{R}^n$, is formulated in $\mathbf{R}^2$. Let $f \in C^2(\overline{\Omega})$ and let $f^j(x,y) := \sum_{p,q\in\Lambda} f((p+c)/2^j, (q+c)/2^j)\phi_p^j(x)\phi_q^j(y)$, $x, y \in \Omega$, $j \in \mathbf{N}$. The constant c is the first moment of the Nth-order Daubechies scaling function ϕ and the index set $\Lambda = \{i \in \mathbf{Z}\colon \operatorname{supp}(\phi_i^j) \cap \Omega \neq \varnothing\}$. Then $\|f - f^j\|_{L^2(\Omega)} \leq C2^{-2j}$ and $\|f - f^j\|_{H^1(\Omega)} \leq C2^{-j}$ where the constant C depends only on the diameter of Ω, the order N and the maximum modulus of the first- and second-order derivatives of f on $\overline{\Omega}$. The interpolation scheme makes basic use of Mallat's algorithm and is used to derive error estimates in terms of wavelet-Galerkin approximate solutions of the elliptic equation $-\Delta u + u = f$ on $\Omega \subset \mathbf{R}^2$, where the Neumann condition $\partial u/\partial n = g$ is imposed over the Lipschitzian boundary $\partial\Omega$.

{For the collection containing this paper see MR1306514}

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