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★**Wavelet matrices and the representation of discrete functions.**

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This article is devoted to the class of $m \times mg$ matrices a_k^j, $j = 0, 1, \cdots, m-1$, $k = 0, 1, \cdots, mg-1$, where $m \geq 2$ is fixed and $g \geq 1$ is arbitrary, whose rows satisfy the orthogonality relations $\sum_k a^j_{k+mr} \overline{a}^{j'}_{k+mr'} = \delta_{jj'}\delta_{rr'}$ and the normalization condition $\sum_k a_k^0 = \sqrt{m}$. They are called wavelet matrices, because they form the raw material for wavelet theory. In fact, under suitable additional conditions there exist compactly supported functions $\psi_j \in L^2(\mathbf{R})$, $j = 0, 1, \cdots m-1$, which satisfy $\psi_j(x) = \sum_k a_k^j \sqrt{m}\psi_0(mx-k)$ such that $\{\psi_0(x-k), m^{l/2}\psi_j(m^l x - k),\ k, l \in \mathbf{Z},\ l \geq 0,\ j = 1, \cdots, m-1\}$ is an orthonormal basis for $L^2(\mathbf{R})$.

A large collection of examples shows that such matrices have been of interest long before the rise of wavelet theory. The authors study the fundamental properties of such matrices. For instance, the simplest wavelet matrices, corresponding to $g = 1$, are completely classified. It is shown how infinite sequences (c_k) can be decomposed with respect to the orthogonal rows of wavelet matrices and their translates. A very important aspect of wavelet matrices is the existence of a highly nontrivial product between wavelet matrices, which endows the set of all wavelet matrices with the structure of an infinite-dimensional Lie group.

{For the collection containing this paper see MR1161244}

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