MR899403 (88i:32042) 32L10 32C37
Haske, Carl (1-RICE); **Wells, R. O., Jr.** **[Wells, Raymond O., Jr.]** (1-RICE)
Serre duality on complex supermanifolds.
Duke Math. J. **54** (1987), *no. 2,* 493–500.

The classical Serre duality theorem states that if M is a compact complex manifold and $\mathcal{F}$ is a locally free sheaf of $\mathcal{O}$-modules on M (in other words, a vector bundle), then there is a natural duality of finite-dimensional complex vector spaces between $H^p(M,\mathcal{F})$ and $H^{n-p}(M,\kappa\otimes\mathcal{F})$, where n is the complex dimension of M and κ is the canonical bundle.

I. B. Penkov [Invent. Math. **71** (1983), no. 3, 501–512; MR0695902] extended this theorem to the category of supermanifolds. The statement is essentially the same except that κ is replaced by $\mathcal{B}er$ the Berezinian sheaf. Penkov's proof uses the theory of $\mathcal{D}$-modules and also gives a version of the duality where $\mathcal{F}$ is allowed to be any coherent sheaf.

In this article, the authors describe an alternative proof which relies on the natural duality between smooth functions and distributions. In other words, they follow J.-P. Serre's original method [Comment. Math. Helv. **29** (1955), 9–26; MR0067489] which works for $\mathcal{F}$ locally free. This supersymmetric version, however, requires a lot of extra careful checking.

Note that for Theorem 8 to be valid one needs to assume, as Serre did, that the $\overline{\partial}$-operator have closed range.

Michael G. Eastwood